

# Holographic cosmic energy, fundamental theories and the future of the universe

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Holographic dark energy models have been recently suggested which most clearly show that accelerating cosmology appears to be incompatible with mathematically consistent formulations of fundamental theories such as string/M theories. In this paper it is however argued that holographic phantom models are no longer incompatible with such theories, provided that we allow for the existence of wormholes and ringholes near the big rip singularity and a quantization condition on the parameter of the equation of state is introduced. It is also seen that such a condition actually implies a quantization of the phantom energy which stabilizes the fluid against decay processes.

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## I. INTRODUCTION

The last years have been really exciting in cosmology. The discovery that the universe is currently accelerating [1] has opened a true plethora of developments which have much attracted researchers from other fields, including string theorists and astrophysicists. Among these developments one cannot forget the implication that Einstein gravity cannot by itself govern the evolution of the universe, which has driven two distinct research avenues. On the one hand, some cosmologists have searched modified (generalized) gravitational theories in which the Hilbert-Einstein action is supplemented with inverse curvature terms able to predict the late time acceleration [2], and on the other hand, others have looked at a scenario where Einstein gravity is preserved but some vacuum fields are introduced to account for the supernova Ia observations [3]. The energy associated with these fields has been dubbed dark energy and, if it exists it would do so in such a way that it becomes currently dominating over all other kinds of energy in the universe. Of particular interest are the so-called quintessence fields [4]. These are equivalent to a cosmic fluid characterized by an equation of state,  $p = w\rho$ , where the energy density  $\rho$  is positive and the index  $w$  has been constrained by observations to be within the interval  $-0.8 \leq w \leq -1.4$  [5]. The resulting negative pressure would then play the role of an anti-gravity regime which is ultimately responsible for the late-time acceleration of the universe. One of the greatest challenges posed by this discovery is its apparent essential incompatibility with those fundamental theories which rely on the existence of a S-matrix or S-vector formulation, that is quantum-gravity and string/M theories [6]. However, more recent work has made great strides toward understanding how de Sitter solutions may be eventually accommodated in a string theory framework, so largely weakening the essential character of the above puzzle, at least in the particular cases being considered. Furthermore, it looks straightforward enough to envisage a universe which accelerates only temporarily, until the dark energy decays e.g via some phase transition, after which it approached Minkowski space where the above puzzle would vanish. Nevertheless, it is also straightforward enough to imagine that the universe will again accelerate on some period of the future where the puzzle would once again reappears. Actually, Caldwell et al [7] have now proposed a model which quite comfortably accommodate current observations where the future phase transition precisely leads to a phantom universe phase. Thus, even though it perhaps hardly could be maintained as being a strictly essential problem, the solution to the above cosmic-fundamental theory puzzle appears still be a necessary requirement.

This cosmic-fundamental theory puzzle -where the word "fundamental" stands here for emphasizing the basic character of the involved field theories- is most apparent when dark energy models are combined with holographic bounds on entropy [8] to yield up a cosmic theory where the short distance UV cutoff and the large distance IR cutoff are mutually related [9]. A holographic model of dark energy has been recently suggested by Li [10] which is based on the following relation between the Hubble parameter and a suitably chosen holographic screen,

$$H^2 = \frac{\dot{a}^2}{a^2} = \frac{8\pi G\rho}{3} = \frac{c^2}{R_h^2}, \quad (1)$$

where  $c$  is a numerical parameter which is related to the index  $w$  by  $w = -(1 + 2/c)/3$ ,  $a \equiv a(t)$  is the scale factor and  $R_h = a \int_t^\infty dt/a(t)$  is the proper size of the future event horizon which is introduced to play the role of the cosmic holographic surface. This integral expression corresponds to the known definition of an event horizon, but its use as a holographic screen makes that definition equivalent through Eq. (1) to the Hubble radius  $H^{-1}$ , in spite of the feature that the actual cosmic horizon can be much larger if an early inflationary period is considered.

Thus, for a scale factor given by  $a = \left( a_0^{1-1/c} + (1 - 1/c)\sqrt{\frac{8\pi G\rho_0}{3}}(t - t_0) \right)^{1/(1-1/c)} = T(t)^{1/(1-1/c)}$ , with  $a_0$  and  $\rho_0$  the scale factor and the energy density, respectively, at the initial time  $t_0$ , in a generic quintessence model, the proper size of the future event horizon is given by [10]

$$R_h = -cT(t)^{1/(1-1/c)} \left( T(t)^{-1/[c(1-1/c)]} \right) \Big|_t^\infty. \quad (2)$$

Clearly, if  $c > 1 \rightarrow w > -1$  then  $R_h = cT(t)$ , which is finite for finite  $t$ . This is the Li proposal [10] for dark energy which becomes well defined only when  $w > -1$ . For such a regime, all the information contained in the universe at time  $t$  would thereby be encoded at the finite surface on the future event horizon. Now, since no information whatsoever on events taking place beyond that horizon could be obtained by any observer at  $t$ , theories based on the S-matrix that relate points infinitely separated spatially could in principle not be consistently formulated within this context.

In this paper we shall discuss in more detail models of holographic phantom cosmology in relation with the possibility of formulating mathematically consistent fundamental theories, extending the above analysis to other dark energy models. It will also be argued that in order for this possibility to be maintained it is necessary to quantize the index of the equation of state,  $p = w\rho$ , and hence the energy of the phantom universe. The resulting quantization of phantom

energy may become a key ingredient for phantom energy stabilization, as it forbids any conversion process involving the emission of arbitrarily small amounts of energy which do not satisfy the quantization requirement.

## II. A HOLOGRAPHIC PHANTOM ENERGY MODEL

A solution to the cosmic-fundamental theory problem could be nevertheless derived if we allow  $w$  to take on values less than  $-1$  [11]. Actually, for  $w < -1$  (i.e. for  $c < 1$ ) it follows from Eq. (2) that the proper size of the future horizon inexorably becomes infinity; or what is equivalent, the future event horizon will vanish for phantom energy, so solving the above puzzle in that case. However, a holographic phantom model cannot use Eq. (1) for the Hubble parameter  $H$  because in this case Eq. (1) leads to  $H = 0$  along the whole cosmic evolution dominated by  $w < -1$ . Thus, the Li proposal for holographic dark energy does not work when  $w < -1$ . For in that case, instead of Eq. (1) I suggest using the following cosmic holographic definition (see also Refs. [12] and [13])

$$H_{ph}^2 = \frac{\dot{a}^2}{a^2} = \frac{8\pi G\rho}{3} = \frac{c^2}{R_*^2}, \quad (3)$$

where  $R_* = a(t) \int_t^{t_*} dt'/a(t') = cT(t)$ , with  $t_*$  the time at which the big rip takes place [12],  $t_* = t_0 + 2a_0^{-3(|w|-1)/2}/[3(|w|-1)]$ . Now,  $H \neq 0$  satisfying the first equality in the previous Eq. (3). So, in the case of the holographic phantom energy we must take for the size of the ultimate region whose total energy is chosen not to exceed the mass of a black hole with the same size,  $L$ , the proper size of the horizon at the big rip  $R_*$ , i.e.  $L = R_*$ , instead of the proper size of the future event horizon  $R_h$ , which was chosen for holographic dark energy with  $c \geq 1$ . Since the cosmic fundamental theory puzzle refers to the future event horizon, we can then have both a well-defined holographic phantom theory and a suitable solution to that puzzle. There is however a question remaining. In the quintessential phantom regime that corresponds to  $w < -1$  there will be a real big rip singularity at the finite time  $t = t_*$  in the future [14]. If we want then to preserve an infinite proper size for the event horizon, it will be later seen that it is necessary to allow [11] for wormhole-mediated connections between the regions before and after the big rip singularity. These traversable tunnels could then be supporting an unbounded flow of information from any point on the region after the big rip to the given observer. Wormholes of different types should actually be expected to occur in general relativity. They in fact correspond to well-defined solutions to the Einstein equations and, therefore, can be considered in our analysis, mainly in models of phantom energy which is actually is the stuff that is required to cast a suitable energy-momentum tensor giving rise to wormhole solution [15,16]. Such wormholes are therefore necessary ingredients in the context of the present paper.

The above usual definition of the event horizon involves the integration limit at  $t = \infty$  and so the entire region defined for times larger than that for the big rip is included in such a definition for phantom energy. Although the big rip corresponds to a curvature singularity and therefore at first sight it would mark the end of the evolution of the universe, there are however at least two reasons why one should also consider the region after the big rip,  $t > t_*$  to be physical. On the one hand, it has been recently shown [16] that wormholes can be made of phantom energy and therefore they should quite naturally crop up in the universe provided that  $w < -1$ . On the other hand and more importantly, it will be later seen that as one approaches the finite future singularity the cosmic spacetime metric becomes expressible as the metric on a five-dimensional hyperboloid which is invariant under the Misner symmetry and therefore describes bounded space-time regions where closed timelike curves can occur through traversable wormholes and ringholes in the neighborhood of the singularity. In both cases, such space-time tunnelings can join the regions before and after the big rip so that information signaling may travel from one region to another without reaching the singularity at finite time which is shortcut in this way. The physical region for all these signalings and the physical objects able to traverse the traversable wormholes and ringholes will then extend beyond the big rip singularity (without passing through it), up to  $t = \infty$ .

First of all we shall investigate how the holographic dark energy scenario may work in the cases of the other contender models for dark energy, namely, the K-essence [17] and the generalized Chaplygin gas [18] frameworks. In the case that instead of quintessence we use K-essence then for the phantom energy regime the scale factor is given by [19]

$$a = a_0 (t - t_*)^{-2\beta/[3(1-\beta)]},$$

where  $a_0$  is an arbitrary initial value for the scale factor,  $t_*$  is the time at the big rip, which is also arbitrary in this model, and  $0 < \beta < 1$ . The proper size of the event horizon would again take on the infinite value

$$R_h = \frac{3(1-\beta)(t-t_*)^{-2\beta/[3(1-\beta)]}}{3-\beta} \left( (t-t_*)^{2\beta/[3(1-\beta)]+1} \right) \Big|_{t=t_*}^{\infty} = \infty. \quad (4)$$

It is therefore possible to find particular sets of K-essence parameters which lead to a scenario where mathematically consistent fundamental theories can be formulated, such as it happens in phantom quintessence. Also similar to the phantom quintessence case, the holographic K-essence screen must also be placed at the scale of the big rip singularity,  $L = R_* = a(t) \int_t^{t_*} dt'/a(t')$ .

We consider next a generalized Chaplygin gas. In this case we have for the scale factor of the universe the relation [20]

$$\dot{a} = Ca \left( A + \frac{B}{a^{3(1+\alpha)}} \right)^{1/[2(1+\alpha)]}, \quad (5)$$

in which  $C = \sqrt{8\pi G/3}$  and  $A$ ,  $B$  and  $\alpha$  are constant whose values generally specify the regime we are working on [20]. It can then be written that

$$R_h = a \int_t^\infty \frac{dt}{a} = a \int_a^\infty \frac{da}{a\dot{a}}. \quad (6)$$

From Eqs. (5) and (6) the size of the event horizon can be generally expressed by means of the integral equation

$$R_h = a \int_a^\infty \frac{da}{Ca^2 \left( A + \frac{B}{a^{3(1+\alpha)}} \right)^{1/[2(1+\alpha)]}}. \quad (7)$$

Exact integration of Eq. (7) cannot be performed for general values of the Chaplygin parameter  $\alpha$ . However, if we choose for the equation of state of the generalized Chaplygin gas the expression  $p = A\rho^{1/3}$  (i.e. taking for  $\alpha$  the value  $-1/3$ ), the above expression can be integrated in closed form to give

$$R_h = a \left( \frac{1}{C^2 \left( A + \frac{B}{a^2} \right)^{1/2}} \right) \Big|_a^\infty = \frac{a}{C^2 B} \left( A^{-1/2} - \left( A + \frac{B}{a^2} \right)^{-1/2} \right). \quad (8)$$

Now, in the regime of the Chaplygin-gas model where the dominant energy condition is preserved [20],  $B > 0$ , it can be seen that the size of the event horizon increases with  $a$  but keeps always a finite value. In the case where the dominant energy condition is violated [20],  $B < 0$ , on the phantom regime, one again obtains the same result, as it can be shown by taking into account that  $a > \sqrt{|B|/A}$  when  $\alpha > -1$ . That behavior must be associated with the feature that Chaplygin models do not lead to a big rip type singularity in the future [20,21]. This result can be generalized to any allowed value of parameter  $\alpha > -1$ . Thus, just like it happens in quintessence models with  $w > -1$  or when  $B > 0$ , the size of the event horizon entering the holographic Chaplygin phantom cosmology is always finite at finite  $t$  and hence, whereas  $R_h$  would always enter the definition of holographic Chaplygin gas Friedmann equation, both for  $B > 0$  and  $B < 0$ , no consistent fundamental theory based on a S-matrix could be constructed in the Chaplygin scenario for any  $B$ .

### III. IS THE EQUATION OF STATE OF A PHANTOM UNIVERSE QUANTIZED?

If the regions before and after the big rip are mutually connected by tunneling circumventing the singularity, then the evolution of an ideal observer in the universe would proceed as follows. It will first evolve in an accelerating universe and sees how this reaches a maximum finite size, then the observer enters e.g. a traversable wormhole by the mouth opening at the expanding region at a time  $t_{enter} < t_*$  and shortcuts the space-time, so circumventing the big rip singularity, to exit out from the wormhole through its mouth opening up into the contracting region at a nonzero time after the big rip ( $t_{exit} > t_*$ ). Thereafter, the observer will see how the universe will steadily contract down to zero as time tends to infinity. For such an observer the evolution of the universe does not reach any singularity at  $t = t_*$  but smoothly goes from any initial time to a infinite time after short cutting the big rip singularity by traversing a wormhole. Mere inspection of the time dependence of the scale factor when  $w < -1$  indicates however that not any value of  $w < -1$  leads to a real value of  $a(t)$ . We now consider in some more detail how the solution to the cosmic-fundamental theory puzzle can be worked out in the quintessential and K-essence phantom models. In the former case, it can be straightforwardly realized that no all values for parameter  $w < -1$  can be continued into the region after the big rip. Indeed, in general all those values of  $w$  which do not satisfy the relation

$$w = -\frac{1}{3} \left( 1 + \frac{2n+3}{n+1} \right), \quad n = 0, 1, 2, \dots, \infty \quad (9)$$

lead to a negative or imaginary scale factor after the big rip.

Generally one would expect the equation-of-state parameter  $w$  to vary smoothly as the dark energy evolves. In the regime where dark energy dominates, this is certainly true when  $w > -1$ , but it is by no means a physical requirement in case that  $w < -1$  as we are going to show in what follows. Condition (9) restricts the phantom models which can allow for a cosmic evolution after the big rip, and therefore the models which can allow for the presence of wormholes and ringholes in the neighborhood of the big rip. In terms of the holographic parameter  $c$  that condition is given by

$$c = \frac{2(n+1)}{2n+3}, \quad n = 0, 1, 2, \dots, \infty \quad (10)$$

We note that the allowed absolute values of  $c$  and  $w$  decrease with  $n$  and tend both to 1 (the cosmological constant case) as  $n \rightarrow \infty$ .

Condition (9) comes from the requirement that the scale factor  $a(t)$  be real and positive also on the region  $t > t_*$  when  $w < -1$ . This does not represent a true *a priori* quantization of the scalar field that makes up phantom energy, but merely a condition that quantizes the equation of state assumed to approximately govern the whole content of the universe in the sense that only some of the possible values of the index of such a equation of state are allowed when the phantom energy largely dominates over all other energies. However, from the definitions of the energy density,  $\rho$ , and pressure,  $p$ , in terms of a scalar field  $\phi(t)$ ,

$$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi) \quad (11)$$

$$p = \frac{1}{2}\dot{\phi}^2 - V(\phi), \quad (12)$$

the condition (9) leads also to a somehow "quantized" phantom field theory given by

$$\phi = i\phi_0 \sqrt{\frac{\rho_0}{3(n+1)D}} \ln a(t) \quad (13)$$

$$V(\phi) = \frac{6n+7}{6(n+1)}\rho_0 a^{1/(n+1)} = \frac{6n+7}{6(n+1)}\rho_0 e^{-i\sqrt{\frac{3D}{(n+1)\rho_0}}\phi/\phi_0}, \quad (14)$$

where  $D = 8\pi G\rho_0/3$  and  $\rho_0$  and  $\phi_0$  are integration constants. This may be viewed as the pre-quantized theory (that is, a theory quantized at a similar level to e.g. Bohr's hydrogen theory) associated with a cosmological complementarity principle, namely that all objects in the phantom universe have two aspects of their existence, on the one hand as members of the cosmic collective which can most generally be characterized by the scale factor  $a$ , and on the other hand as local individuals in the universe and hence can be made to depend on the scalar field  $\phi$ . Pairs of numbers are therefore necessary to define the value of each dynamic physical variable describing the state of the phantom universe. The algebra of complex numbers may take care of this, whereby the real component is always associated with the collective aspect ( $a$ ) and the imaginary component is associated with the individualized aspect ( $\phi$ ). This would ultimately provide with an explanation to the hitherto unexplained fact that the kinetic field term  $\dot{\phi}^2$  is definite negative for phantom energy, and would restrict the occurrence of wormholes and ringholes on only the neighborhood of the big rip singularity. In fact, it is well known that the catastrophic creation of particles that concentrate on the chronology horizon near their throat renders these tunnels unstable [22], so that all those wormholes and ringholes naturally created in a phantom universe would be unstable except on regions near the big rip where the created particles cease to have any individualized behavior to all adhere to the collective ripping apart produced by the super-accelerated expansion that breaks down all forces leading to any concentration of such particles on the chronology horizon.

The allowed proper sizes of the future event horizon which satisfy the above condition (9) are given by

$$R_h = -\frac{2(n+1)}{2n+3}T(t)^{-2(n+1)}(T(t')^{2n+3})|_t^\infty. \quad (15)$$

Relative then to an observer at a time  $t < t_*$ , since the power of  $T(t')$  is definite odd and  $T(t')$  is definite negative after  $t_*$ ,  $R_h = \frac{2(n+1)}{2n+3}T(t) + \infty$ . As the observer approaches the big rip from  $t < t_*$ ,  $R_h$  will remain being  $+\infty$  and  $\dot{R}_h = -(2n+3)^{-1}$ , and at  $t = t_*$ , where  $T(t) = 0$ ,  $R_h = +\infty$  as well. For  $t > t_*$ , we will have  $R_h = +\infty - \frac{2(n+1)}{2n+3}|T(t)|$  which keeps being infinity, up to  $t \rightarrow \infty$  at which asymptotic situation  $R_h$  shrinks to zero, just like it does in case that

$c > 1$ . It follows that if the condition (9) is satisfied then the proper size of the future horizon does not shrink from its infinite value, all the way, even at  $t = t_*$ , up to  $t = \infty$ , at which point it vanishes. In the case of the holographic phantom model, it can be seen that the evolution of the universe along time  $t < t_*$  will progress so that the size of  $R_*$  shrinks down to just the size of a wormhole or ringhole mouth (which then measures the IR cutoff) that exceeds the UV cutoff which should necessarily be placed at the scale of the hole throats, or less. This implies that the IR cutoff will always remain larger than the UV cutoff at any finite time in the future and therefore the definition of holographic phantom energy is kept intact if condition (9) holds. Hence, the global argument against holographic phantom raised by Huang and Li [23] no longer applies if condition (9) is satisfied. On the other hand, neither the argument by the same authors [23] of a decreasing entropy could be adduced against holographic phantom. In fact, if the entropy of the universe when only holographic phantom energy is present is given by

$$S_{ph} = \pi M_p^2 R_*, \quad (16)$$

then the feature that  $\dot{R}_h = c - 1 = -(2n + 3)^{-1} < 0$  implies that  $S_{ph}$  decreases as  $t$  goes on. However, since the temperature of the phantom stuff is definite negative [24], processes for which the entropy decreases would be physically allowed. All the above discussion can also be applied to the case of phantom K-essence by taking into account that in this case the condition that the parameter  $\beta$  should satisfy is

$$\beta = \frac{3n}{3n + 1}, \quad n = 1, 2, 3, \dots \quad (17)$$

with  $n$  necessarily being finite as  $\beta < 1$ . Using Eq. (17) we in fact recover  $R_h = +\infty$  and obtain

$$R_* = \frac{t_* - t}{2n + 1},$$

for the K-essential phantom model.

#### IV. CIRCUMVENTING THE BIG RIP WITH WORMHOLES

The solution to the cosmic puzzle for fundamental theories provided by phantom energy has been already shown to require a connection between the regions before and after the big rip which can be thought to be implemented by means of Lorentzian wormholes and ringholes [15]. Such a tunneling process would lead to a multiply connected effective cosmic space-time ranging from  $t = 0$  to  $t = \infty$ , with the holes taking place everywhere on both sides of the big rip singularity which is thereby avoided by any signaling. Actually, wormholes and ringholes are quite natural objects in the presence of phantom energy, a situation where the dominant energy condition is violated. In that situation one would expect the phantom stuff itself to make the exotic material required for these tunnels to avoid pinching off and be really traversable [16]. In the neighborhood of the big rip, i.e. on the region where the tunnels are stable even quantum-mechanically,  $T$  becomes so small that we can in fact take

$$a(t) = \frac{1}{T^{2(n+1)}} \simeq \frac{a_0}{\sin(T^{2(n+1)}/a_0)} = \frac{a_0}{\sin x}, \quad (18)$$

so that in the considered region the metric becomes

$$ds^2 = -dt^2 + \frac{a_0^2}{\sin^2 x} d\Omega_3^2, \quad (19)$$

with  $d\Omega_3^2$  the metric on the unit three-sphere. This space-time can also be visualized as a five-hyperboloid defined by

$$-x_0^2 + \sum_{j=1}^4 x_j^2 = a_0^2. \quad (20)$$

This hyperboloid can be embedded in  $E^5$  so that the most general approximate expression for the metric of the phantom accelerating space in the neighborhood of the big rip is then that which is induced in this embedding, that is,

$$ds^2 = -dx_0^2 + \sum_{j=1}^4 dx_j^2, \quad (21)$$

whose topology is  $R \times S^4$  and invariance group can be approximated by the group  $SO(4,1)$ , showing ten Killing vectors (four boosts and six rotations).

Metric (21) can be exhibited in a not still static form by introducing the set of specific coordinates  $x \in (0, \infty)$ ,  $\Psi_3, \Psi_2 \in (0, \pi)$ ,  $\Psi_1 \in (0, 2\pi)$ ,

$$\begin{aligned} x_4 &= a_0 \sin \Psi_3 \sin \Psi_2 \cos \Psi_1 \\ x_3 &= a_0 \sin \Psi_3 \sin \Psi_2 \sin \Psi_1 \\ x_2 &= a_0 \sin \Psi_3 \cos \Psi_2 \\ x_1 &= a_0 \frac{\cos \Psi_3}{\sin x} \\ x_0 &= a_0 \frac{\cos \Psi_3 \cos x}{\sin x} \end{aligned} \tag{22}$$

In terms of these coordinates, the above metric becomes

$$ds^2 = - (1 - a_0^{-2} r^2) \frac{a_0^2 C^2 \rho_0 dt^2}{T(t)^2} + \frac{dr^2}{(1 - a_0^{-2} r^2)} + r^2 d\Omega_2^2, \tag{23}$$

in which we have taken  $r = a_0 \sin \Psi_3$  and  $d\Omega_2^2$  is the metric on the unit two-sphere. This metric shows both the apparent singularity at an event horizon at  $r = a_0$  and the curvature singularity at the big rip  $t = t_*$ . The latter singularity can however be removed from the metric, so converting this into a static metric, if we redefine time so that it will cover the entire interval from  $-\infty$  to  $+\infty$ . With the new time coordinate

$$\tau = \ln \left[ \left( \frac{T(t)}{T_0} \right)^{-2(n+1)} \right], \tag{24}$$

where  $T_0$  is an integration constant, we in fact get a static metric having exactly the form of the De Sitter line element:

$$ds^2 = - (1 - a_0^{-2} r^2) d\tau^2 + \frac{dr^2}{(1 - a_0^{-2} r^2)} + r^2 d\Omega_2^2, \tag{25}$$

in the neighborhood of the big rip singularity. Now, in order to exhibit that region as a multiply connected region due to the presence of wormholes and ringholes, we again visualize the space-time by the five-hyperboloid (20), with embedding metric (21), exhibiting it by means of coordinates (22), but with  $x_1$  and  $x_0$  replaced for

$$x_1 \rightarrow a_0 \cos \Psi_3 \cosh(\tau/a_0), \quad x_0 \rightarrow a_0 \cos \Psi_3 \sinh(\tau/a_0). \tag{26}$$

One can then show that at least in the close neighborhood of the big rip singularity the phantom space-time can be made to have a multiply connected topology because a symmetry like that is satisfied by the Minkowskian covering to Misner space is holding on that region. In fact, on the Minkowskian five-hyperboloid visualizing de Sitter space such a symmetry can be expressed by the identifications

$$(x_0, x_1, x_2, x_3, x_4) \leftrightarrow (x_0 \cosh(mb) + x_1 \sinh(mb), x_1 \cosh(mb) + x_0 \sinh(mb), x_2, x_3, x_4), \tag{27}$$

where  $b$  is a dimensionless arbitrary constant and  $m$  is any integral number. There will be then a boost transformation in the space described by metric (25) which is implied by the boost transformation in the  $(x_0, x_1)$ -plane driven by the above identification. It can be checked that metric (19) is invariant under symmetry (27) in the region covered by that metric defined by  $x_1 > |x_0|$ , with boundaries at  $x_1 = \pm x_0$  (which correspond to the big rip  $t = t_*$  and  $t = \infty$ ) and  $x_4 + x_3 + x_2 = a_0^2$  (i.e. the event horizon at  $r = a_0$  on the space-time with metric characterized by time  $\tau$ ), provided  $\tau \rightarrow \tau + mb$ . These boundaries describe the chronology horizons for the region around the big rip. Now, since a chronology horizon generally describes the onset of a non-chronal region filled with closed timelike curves (CTC's), a region which on the coordinates defined in terms of time  $\tau$  goes from  $\tau = -\infty$  to  $\tau = +\infty$ , or vice versa, inside the

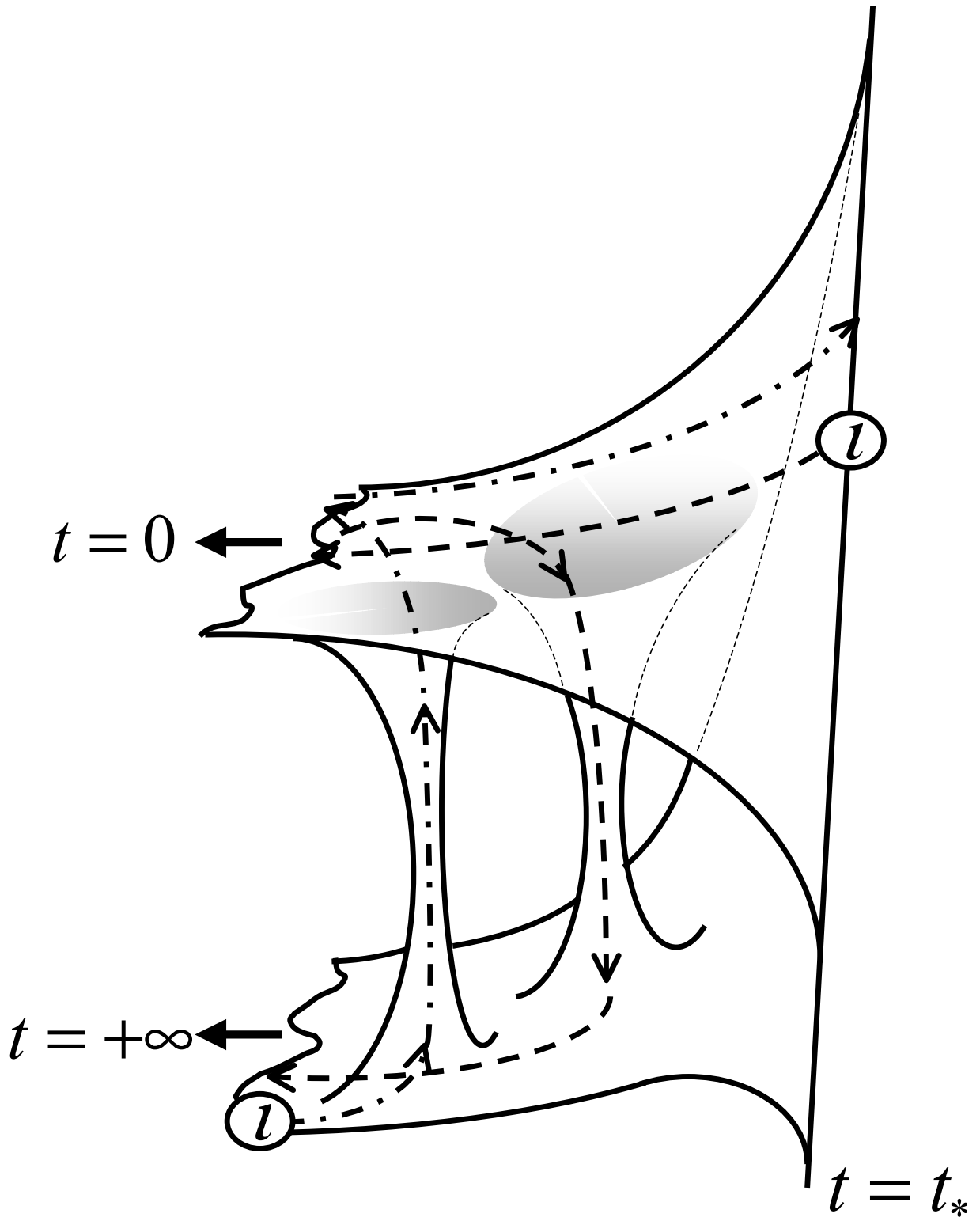


FIG. 1: Pictorial representation of the possible trajectories that signalings may follow through wormholes connecting the space-time regions before and after the big rip singularity. The  $l$ s on the figure label the origin of the signals.

event horizon, would reflect into an also non-chronal region for coordinates defined in terms of time  $t$  whose onset is either at  $t = +\infty$  to go continuously down to  $t = t_*$ , after passing through  $t = 0$ , or at  $t = t_*$  to continuously go up to  $t = +\infty$ , after passing through  $t = 0$  as well. The existence of CTCs in the non-chronal region can be implemented by the presence of naturally existing grown up [21] wormholes and ringholes which then could, according to the above discussion, mutually connect the non-chronal regions before and after the big rip, such as it has been depicted in Fig. 1.

For phantom cosmological models which do not satisfy condition (9) [or (10)] the connection between both sides of the singularity could not be made as the universe would cease to exist as a real entity, just after the big rip. In such a case, the largest possible proper size of the future horizon would be given by  $R_* = cT(t) < R_h (c \geq 1)$ , with  $c \neq \frac{2(n+1)}{2n+3}$ , which is necessarily finite, so keeping or even aggravating the puzzle on fundamental theories. We may therefore conclude that if we want to have an accelerating cosmology which be free from the critical problems posed to the definition of fundamental theories by the existence of a finite horizon in the future, then we have to recourse to a phantom quintessence or K-essence model restricted by the condition (9) or (17). We notice that that condition in turn implied a minimal value for  $w$  of  $w_{min} = -4/3$  and actually a quantization of the index of the equation of state of the universe, according to which any feasible tracking model of the universe with a future big rip singularity would undergo an evolution that proceeded by steps and be in this way governed by some still unexplored quantum rules.

Avoiding the big rip by tunneling through wormholes does not actually contradict the idea that the holographic surface be placed at the big rip as it could at first sight seem. In fact, according to the covariant definition of a holographic surface [25], the horizon at the big rip would correspond to an optimal holographic screen both in the absence of closed timelike curves near the singularity and in the presence of wormholes around that singularity, as in both cases this surface marks the place where the generating geodesics cease to show expansion. If there are not closed timelike curves the holographic big rip surface would represent the end of the universe, but there will be a contracting region after that surface if closed timelike curves are allowed to occur near it. This can be better seen on the corresponding Penrose-Carter diagrams (see Fig. 2). On these diagrams, it can be shown that, according to the Bousso's covariant holographic formulation [25], the holographic optimal screen is always placed on the big rip, no matter whether or not there are wormholes on both sides of the singularity.

The above possibility to avoid the big rip actually involves all observers in the universe because of the following two reasons. (1) Phantom energy is expected to largely dominate on the neighborhood of the big rip singularity, and therefore wormholes having as their exotic stuff energy that phantom energy should crop up everywhere on that neighborhood; and (2) the Bekenstein information-energy bound [26],

$$I < I_{max} = \frac{2\pi ER}{\hbar c \log 2},$$

from which the holographic principle truly originates, would dictate that for a phantom universe with wormholes around the big rip the spherical radius [12]  $R = a(t) \int_t^{t_{max}} dt' / a(t') = \infty$ , and hence  $I_{max} = \infty$ , that is there exists no restriction on the amount of information that can be transferred through the wormholes from one side of the big rip singularity to the other.

It is worth noticing that the existence of CTCs in the neighborhood of the big trip hypersurface on both sides of  $t = t_*$ , may extend and modify the Bousso's definition of that hypersurface as a holographic surface [25]. Actually, screens, preferred screens and optimal screens are geometrical concepts which can only be defined in a causal space-time. If we would not allow CTCs to occur then the surface at the big rip, let's denote it by  $B$ , corresponded to a preferred screen in the Bousso's sense, as the geodesic generators of it in the accelerated expansion ceases to have positive expansion and turns to be negative precisely at the big rip. Nevertheless, when CTCs are allowed to occur there will also be null geodesics which turn from positive to negative expansion before or after reaching surface  $B$ , albeit very near to it. This would ultimately leads to the notion of preferred or optimal screens having a given width, and hence of a D-dimensional holographic bound where the information is encoded not just on a D-1 dimensional surface but on a D dimensional volume with small width.

## V. FURTHER COMMENTS

The question now is, does this quantization of the equation of state imply any quantization for the phantom energy?. Even though the absence of asymptotic flatness in the cosmological phantom space precludes a meaningful definition of energy for that space, one may still recourse to its thermodynamic properties to obtain an estimate of the phantom energy. In fact, from the general definition of temperature and entropy for a dark energy fluid [24],  $T = \kappa(1+w)a^{-3w}$ ,  $S = C_0 (T/(1+w))^{1/w} V$ , where  $\kappa$  and  $C_0$  are positive constants and  $V$  is the volume being considered, we can

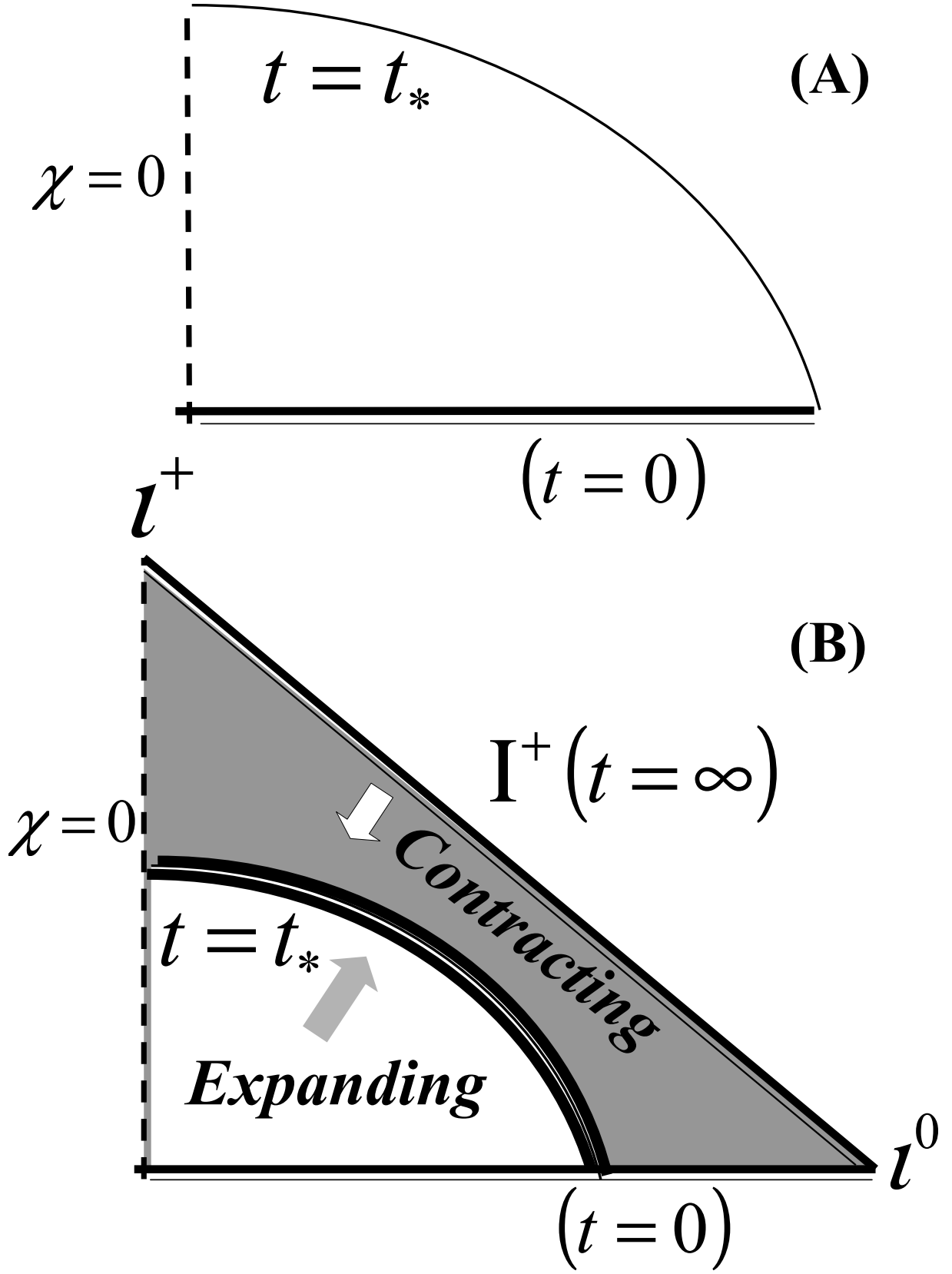


FIG. 2: Penrose-Carter diagrams and their corresponding holographic screen for a phantom universe with (A) no wormholes and (B) wormholes which continuously branch off from and to the space-time regions very near to the big rip singularity. The arrows indicate the direction of geodesics with positive expansion. In case (B) the screen surface becomes a screen volume (see the text).

estimate

$$E \simeq ST = \kappa^{(1+w)/w} C_0 (1+w) a^{-3(1+w)} V. \quad (28)$$

For the phantom regime,  $w < -1$ , we then have a negative quantized energy given by

$$E_n = -\frac{C(n)}{3(n+1)} a^{1/(n+1)} V, \quad (29)$$

where the constant  $C(n)$  is given by

$$C(n) = \gamma C_0 \kappa^{3/(3n+4)}, \quad (30)$$

with  $\gamma$  a numerical constant. For a box with given volume  $V$  filled only with phantom energy at a given time  $t$ , the negative phantom energy is bounded from above for  $n = \infty$ , at  $E_\infty = 0$ . Moreover, since  $T$  and  $E_n$  are both definite negative, the population probability law will have the usual Boltzmann dependence,  $\exp(-|E(n)|/k_b|T|)$ , and there will be no population inversion. If we immerse a given matter level-system  $\sigma$  into the box filled with phantom energy, since the energy of the system is positive, then any energy exchange between the system  $\sigma$  and the phantom fluid would inexorably lead to a decrease of the energy of  $\sigma$ , and hence the energy of that system will be also bounded from above. We see therefore that the conditions for the existence of a negative temperature (that is the quantum nature of the whole system and energy boundedness from above) are satisfied, contrary to the claim in Ref. 27.

Even more interesting is the fact that at a given  $t$  any amount of phantom energy enclosed in a volume  $V$  can only change by jumps, the energy exchanged in each jump  $n \rightarrow n-1$  being given by

$$\Delta E_{ph} = \frac{(n+1)C(n-1)a^{1/n} - nC(n)a^{1/(n+1)}}{3n(n+1)} V. \quad (31)$$

Thus, processes such as the decay of phantom fields into gravitons which were considered to be responsible for the violent instabilities of the phantom energy [28] can no longer take place, so that phantom energy is rendered stable. In this way, although the phantom shows rather weird properties, none of them seems to be precluding its existence in a cosmological context where most observations really point to values of  $w$  less than -1. The almost desperate attempts to relax the observed values of parameter  $w$  from  $w < -1$  to  $w > -1$  by invoking mimicking illusion effects [29,30] or the conversion of photons emitted by SNe Ia into ultra-light axions [31] have proved not to be enough in magnitude or generality (by defect or excess) to justify their purposes. After all, if phantom energy is quantized one could expect most, if not all, of its "bad" properties to be justified.

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